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A NO-SLIP INTERFACE CRACK IN A SHEAR FIELD

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Abstract—Solutions are given for an interface crack having no-slip zones near the crack tips and whose surfaces are loaded in pure shear and combined tension-shear. The interface crack is found to open at two places with part of the smooth region in contact, provided that the tension/shear ratio is small enough. The crack tip stress-intensity factors are related by the Dundurs' constant. Numerical calculations were performed with material constants which pertain to an example of the cancellous bone/PMMA interfaces.

INTRODUCTION

One of the main concerns in designing composite structures is the possibility of interfacial failure between the comprising materials. Because of the increasing use of these structures in various fields of engineering, interface fracture mechanics has become an important research subject. The stresses around a crack lying between two dissimilar elastic media was first studied by Williams in 1959[1] and later by England[2] and Erdogan[3]. These conventional treatments have yielded results which are physically unrealistic, namely material overlapping at the crack tip. Recently, Comninou showed that such unphysical material overlapping can be removed by assuming that the two crack surfaces are in smooth contact at the crack tip regions[4]. However, we note in many actual engineering applications, interfaces are very rough and even interdigitated. The presence of such interdigitations can prevent relative slip at the interface or a portion thereof and thus enable shear loads to be sustained there even when the interface has failed in tension. Investigations of such a no-slip interface crack, both in the interior and on the edge, have been solved under pure tension[5, 6]. This paper is a continuation of that series of studies. It presents a solution for such an internal, interfacial crack under action of pure shear and also for combined loading which is still predominantly shear.

The solution is motivated by problems in internal artificial human joints, in which the artificial material forming the articulating surface is connected to the bone by room temperature hardening polymer, polymethylmethacrylate (PMMA). A significant problem with the artificial joints is that they can become loose from the bone. Although the exact mechanism of loosening is as yet unknown, the process probably involves mechanical crack propagation along the cancellous bone-PMMA interface[7]. A hypothesized loosening process is, first, propagation of initial interface flaws to complete interface failure, next growth of fibrous material at the interface due to relative motion, and finally degradation of the lining material to cause joint pain and symptoms of clinical loosening. The first step in establishing the validity of this hypothesis is, therefore, the study of the crack propagation phenomena and the mechanics of interface cracks. Cancellous bone is spongelike in appearance with pore sizes of the order of 1 mm. Because of this structure, the bone-PMMA interface is very irregular, with typical perturbations of greater than 1 mm. From the finite element studies of these total joint systems, a certain part of this interface is subject to rather high shear[8, 9]. Examples are the stem regions in the hip and the pole regions in the knee. This paper is concerned about how a crack propagates along the interface in those regions where the interfacial stress field is dominated by shear. Consequently, with this specific application in mind, the material constants used in the

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numerical calculation for this solution are taken to be those of cancellous bone and PMMA. However, such a no-slip interface crack may also be physically realistic in other applications where crack surfaces are very rough.

1. PURE SHEAR

Let the interface crack be represented by a line segment $(-a, b)$ on the x-axis, consisting of a smooth region $(-1, 1)$ and two interlocked regions $(-a, -1)$ and $(1, b)$, as shown in Fig. 1. The two interlocked regions are assumed to have failed in the tensile direction; but due to the interfacial interlocks, can still sustain shear loads. We will show in this section that for such an interface crack, opening can occur under pure shear loading. Similar phenomenon also occurs in Comninou's interface crack having smooth contact zones near the tips[10].

If one assumed first that an interface crack with no-slip tips did not open at all under shear loading, then one would find tensile contact stresses over half of the smooth region. In an attempt to eliminate this problem, the crack was allowed to be opened in the region $(-a, \gamma)$ but closed in (γ, b), where γ is an unknown parameter to be determined in the solution process by requiring that the crack is smoothly closed at $x = \gamma$. The problem of tensile contact stresses in the smooth region was eliminated in this case; however, the stress in the interlocked region $(1, b)$ was then found to become tensile, which did not agree with our initial assumption. To relieve this problem, the region $(1, b)$ was allowed also to open (Fig. 2). The solution obtained met all the physically required inequality conditions. This solution is presented in the following.

Problem formulation

The boundary conditions are as follows:

$$\sigma_{yy} = 0, \quad \Delta u_x = 0, \quad \Delta u_y \geq 0, \quad -a < x < -1, \; 1 < x < b, \tag{1a}$$

$$\sigma_{yy} = 0, \quad \sigma_{xy} = 0, \quad \Delta u_y \geq 0, \quad -1 < x < \gamma, \tag{1b}$$

$$\sigma_{xy} = 0, \quad \Delta u_y = 0, \quad \sigma_{yy} \leq 0, \quad \gamma < x < 1, \tag{1c}$$

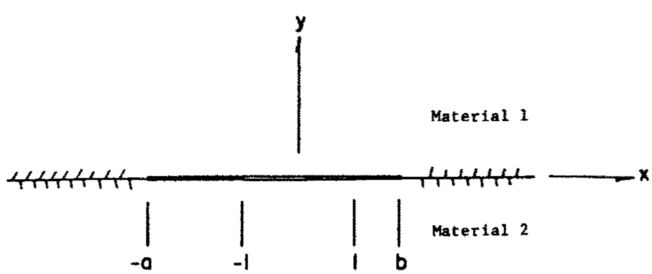

Fig. 1.

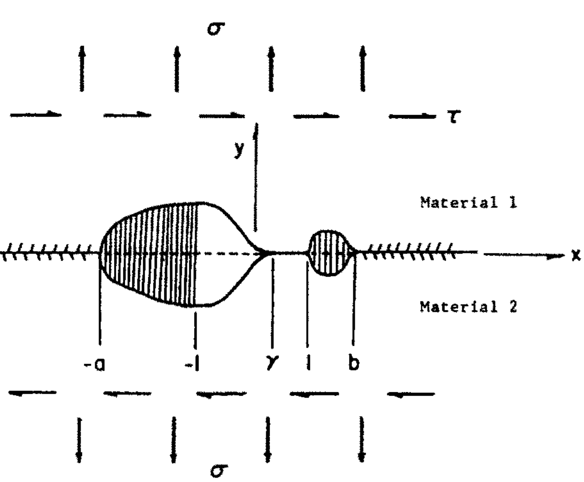

Fig. 2.

where we define,

$$\alpha = \frac{\mu_1(\kappa_2+1) - \mu_2(\kappa_1+1)}{\mu_1(\kappa_2+1) + \mu_2(\kappa_1+1)} \tag{2a}$$

$$\beta = \frac{\mu_1(\kappa_2-1) - \mu_2(\kappa_1-1)}{\mu_1(\kappa_2+1) + \mu_2(\kappa_1+1)} \tag{2b}$$

$$C = \frac{2\mu_2(1+\alpha)}{(\kappa_2+1)(1-\beta^2)} \tag{2c}$$

$$B_x(x) = \frac{\partial}{\partial x}[u_x^1(x,0) - u_x^2(x,0)] \quad -1<x<1, \tag{3a}$$

$$B_1(x) = \frac{\partial}{\partial x}[u_y^1(x,0) - u_y^2(x,0)] \quad -a<x<\gamma, \tag{3b}$$

$$B_2(x) = \frac{\partial}{\partial x}[u_y^1(x,0) - u_y^2(x,0)] \quad 1<x<b. \tag{3c}$$

Here, $\kappa = 3-4\nu$ for conditions of plane strain, $\kappa = (3-\nu)/(1+\nu)$ for plane stress, and α, β are the Dundurs' constants (see [11]).

Following the formulation procedures as in the earlier paper[5], the interfacial conditions (1) lead to

$$\tau - C\left\{\beta B_1(x)H(\gamma - x) - \frac{1}{\pi}\int_{-1}^{1}\frac{B_x(\xi)}{\xi - x}\,d\xi\right\} = 0, \quad -1<x<1, \tag{4}$$

$$\beta B_x(x)H(x+1) + \frac{1}{\pi}\int_{-a}^{\gamma}\frac{B_1(\xi)\,d\xi}{\xi - x} + \frac{1}{\pi}\int_{1}^{b}\frac{B_2(\rho)\,d\rho}{\rho - x} = 0, \quad -a<x<\gamma, \tag{5}$$

$$\frac{1}{\pi}\int_{-a}^{\gamma}\frac{B_1(\xi)\,d\xi}{\xi - x} + \frac{1}{\pi}\int_{1}^{b}\frac{B_2(\rho)\,d\rho}{\rho - x} = 0, \quad 1<x<b. \tag{6}$$

To insure single-valued displacements, the following additional conditions are also imposed.

$$\int_{-1}^{1} B_x(\xi)\,d\xi = 0, \tag{7a}$$

$$\int_{-a}^{\gamma} B_1(\xi)\,d\xi = 0, \tag{7b}$$

$$\int_{1}^{b} B_2(\xi)\,d\xi = 0. \tag{7c}$$

From eqns (4) and (7a)

$$B_x(x) = \frac{1}{(1-x^2)^{1/2}}\left[\frac{-\tau x}{C} - \frac{\beta}{\pi}\int_{-1}^{\gamma}\frac{B_1(\xi)(1-\xi^2)^{1/2}\,d\xi}{\xi - x}\right], \quad -1<x<1. \tag{8}$$

Substituting (8) into (5),

$$\frac{1}{\pi}\int_{-a}^{\gamma}\frac{B_1(\xi)\,d\xi}{\xi - x} + \frac{1}{\pi}\int_{1}^{b}\frac{B_2(\rho)\,d\rho}{\rho - x} - \frac{\beta^2}{\pi}\frac{H(x+1)}{(1-x^2)^{1/2}}\int_{-1}^{\gamma}\frac{B_1(\xi)(1-\xi^2)^{1/2}\,d\xi}{\xi - x}$$
$$= \frac{\beta\tau x}{C(1-x^2)^{1/2}}H(x+1), \quad -a<x<\gamma. \tag{9}$$

Replacing x in (6) with z and defining the following coordinate transformations,

$$s = (x - \Gamma_2)/\Gamma_1, \quad \zeta = (\xi - \Gamma_2)/\Gamma_1$$
$$w = (z - \lambda_2)/\lambda_1, \quad \phi = (\rho - \lambda_2)/\lambda_1 \tag{10}$$

where

$$2\Gamma_1 = \gamma + a, \quad 2\Gamma_2 = \gamma - a$$
$$2\lambda_1 = b - 1, \quad 2\lambda_2 = b + 1, \tag{11}$$

eqn (6) becomes

$$\frac{\Gamma_1}{\pi}\int_{-1}^{1}\frac{\bar{B}_1(\zeta)\,d\zeta}{(\Gamma_1\zeta+\Gamma_2-\lambda_1 w-\lambda_2)}+\frac{1}{\pi}\int_{-1}^{1}\frac{\bar{B}_2(\phi)\,d\phi}{\phi-w}=0, \quad -1<w<1 \tag{12}$$

and eqn (9) becomes

$$\frac{(1-\beta^2)}{\pi}\int_{-1}^{1}\frac{\bar{B}_1(\zeta)\,d\zeta}{\zeta-s}+\frac{\lambda_1}{\pi}\int_{-1}^{1}\frac{\bar{B}_2(\phi)\,d\phi}{(\lambda_1\phi+\lambda_2-\Gamma_1 s-\Gamma_2)}$$
$$\frac{-\beta^2}{\pi}\int_{-1}^{1}\frac{\bar{B}_1(\zeta)\,d\zeta}{(\zeta-s)}\left\{\left[\frac{1-(\Gamma_1\zeta+\Gamma_2)^2}{1-(\Gamma_1 s+\Gamma_2)^2}\right]^{1/2}H(\Gamma_1 s+\Gamma_2+1)\right.$$
$$\left.\times H(\Gamma_1\zeta+\Gamma_2+1)-1\right\}d\phi$$
$$=\frac{\beta\tau}{C}\cdot\frac{(\Gamma_1 s+\Gamma_2)}{[1-(\Gamma_1 s+\Gamma_2)^2]^{1/2}}H(\Gamma_1 s+\Gamma_2+1), \quad -1<s<1. \tag{13}$$

The remaining conditions are:

$$\int_{-1}^{1}\bar{B}_1(\zeta)\,d\zeta=0, \tag{14a}$$

$$\int_{-1}^{1}\bar{B}_2(\phi)\,d\phi=0. \tag{14b}$$

Solution

Assuming $B_1(\zeta)$ and $B_2(\zeta)$ have integrable singularities at both of their end points, we write

$$\bar{B}_1(\zeta)=b_1(\zeta)/(1-\zeta^2)^{1/2}, \tag{15a}$$

$$\bar{B}_2(\phi)=b_2(\phi)/(1-\phi^2)^{1/2}, \tag{15b}$$

writing eqns (12)–(14) in discrete form, with the Gauss–Chebshev integration formula[12], we have:

$$\sum_{i=1}^{n}\frac{1}{n}\left\{\frac{\Gamma_1 b_1(\zeta_i)}{(\Gamma_1\zeta_i+\Gamma_2-\lambda_1 w_k-\lambda_2)}+\frac{b_2(\phi_i)}{\phi_i-w_k}\right\}=0. \tag{16}$$

$$\sum_{i=1}^{n}\frac{1}{n}\left\{\left[\frac{(1-\beta^2)}{(\zeta_i-s_k)}-\beta^2K(\zeta_i,s_k)\right]b_1(\zeta_i)+\frac{\lambda_1 b_2(\phi_i)}{(\lambda_1\phi_i+\lambda_2-\Gamma_1 s_k-\Gamma_2)}\right\}$$
$$=\frac{\beta\tau}{C}\cdot\frac{(\Gamma_1 s_k+\Gamma_2)}{[1-(\Gamma_1 s_k+\Gamma_2)^2]^{1/2}}H(\Gamma_1 s_k+\Gamma_2+1). \tag{17}$$

$$\sum_{i=1}^{n}\frac{b_1(\zeta_i)}{n}=0, \tag{18a}$$

$$\sum_{i=1}^{n}\frac{b_2(\phi_i)}{n}=0. \tag{18b}$$

where

$$K(\zeta_i,s_k)=\frac{1}{\zeta_i-s_k}\left\{\left[\frac{1-(\Gamma_1\zeta_i+\Gamma_2)^2}{1-(\Gamma_1 s_k+\Gamma_2)^2}\right]^{1/2}H(\Gamma_1 s_k+\Gamma_2+1)\cdot H(\Gamma_1\zeta_i+\Gamma_2+1)-1\right\}, \tag{19a}$$

$$\zeta_i=\phi_i=\cos\left(\pi\frac{2i-1}{2n}\right), \quad i=1,\ldots,n \tag{19b}$$

$$s_k=w_k=\cos\left(\frac{\pi k}{n}\right), \qquad k=1,\ldots,n-1. \tag{19c}$$

We further require that the contact pressure at $x=\gamma$ must vanish[13], which leads to

$$\frac{C}{\pi}\left[\int_{-1}^{1}\frac{b_2(\phi)\,\mathrm{d}\phi}{(\phi-\lambda^*)(1-\phi^2)^{1/2}}+\int_{-1}^{1}\frac{b_1(\zeta)\,\mathrm{d}\zeta}{(\zeta-1)(1-\zeta^2)^{1/2}}\right]+\frac{1}{[1-(\Gamma_1+\Gamma_2)^2]^{1/2}}\times$$
$$\left\{-\beta\tau(\Gamma_1+\Gamma_2)\frac{-\beta^2 C}{\pi}\int_{-1}^{1}\frac{b_1(\zeta)}{(1-\zeta^2)^{1/2}}\cdot\frac{[1-(\Gamma_1\zeta+\Gamma_2)^2]^{1/2}}{(\zeta-1)}\,\mathrm{d}\zeta\right\}=0. \tag{20}$$

where $\lambda^*=(\gamma-\lambda_2)/\lambda_1$.

During the process of solution, "a" and "b" were first given. Equations (16)–(19) were subsequently solved for a series of assumed values of γ. The correct solution was then taken as the one that also satisfied eqn (20). The normal traction at the contact zone as well as the relative displacements at the separation zones were calculated to ensure that the inequality conditions were met.

Results

It was found that eqns (16)–(20) can be satisfied by more than one γ. However, of these solutions, only one of them met all the inequality conditions as required physically. Once the solution was found for a given set of "a" and "b", the stress intensity factors can be obtained as follows.

$$\underset{x=-a^+}{K_I}=\operatorname*{Lim}_{s\to-1^+}\left\{(-s-1)^{1/2}\frac{C}{\pi}\int_{-1}^{1}\frac{b_1(\zeta)\,\mathrm{d}\zeta}{(\zeta-s)(1-\zeta^2)^{1/2}}\right\}=\frac{-C}{\sqrt{2}}b_1(-1) \tag{21a}$$

$$\underset{x=-a^-}{K_{II}}=\operatorname*{Lim}_{s\to-1^-}\left\{(s+1)^{1/2}(-C)\beta\frac{b_1(s)}{(1-s^2)^{1/2}}\right\}=\frac{-C}{\sqrt{2}}\beta b_1(-1) \tag{21b}$$

$$\underset{x=-1^+}{K_{II}}=\operatorname*{Lim}_{x\to-1^+}\left\{(-x-1)^{1/2}\left[\frac{C}{\pi}\int_{-1}^{1}\frac{b_x(\xi)\,\mathrm{d}\xi}{(\xi-x)(1-\xi^2)^{1/2}}\right]\right\}=\frac{-C}{\sqrt{2}}b_x(-1) \tag{21c}$$

$$\underset{x=1^-}{K_I}=\operatorname*{Lim}_{x\to1^-}\left\{(1-x)^{1/2}\left[C\beta\frac{b_x(x)}{(1-x^2)^{1/2}}\right]\right\}+\operatorname*{Lim}_{w\to-1^+}\left\{(-1-w)^{1/2}\frac{C}{\pi}\int_{-1}^{1}\frac{b_2(\phi)\,\mathrm{d}\phi}{(\phi-w)(1-\phi^2)^{1/2}}\right\}$$
$$=\frac{C\beta}{\sqrt{2}}b_x(1)-\frac{C}{\sqrt{2}}b_2(-1) \tag{21d}$$

$$\underset{x=1^+}{K_{II}}=\operatorname*{Lim}_{x\to1^+}\left\{(x-1)^{1/2}\frac{C}{\pi}\int_{-1}^{1}\frac{b_x(\xi)\,\mathrm{d}\xi}{(\xi-x)(1-\xi^2)^{1/2}}\right\}+\operatorname*{Lim}_{w\to-1^-}\left\{(1+w)^{1/2}(-C)\beta\frac{b_2(w)}{(1-w^2)^{1/2}}\right\}$$
$$=\frac{-C}{\sqrt{2}}b_x(1)-\frac{C\beta}{\sqrt{2}}b_2(-1) \tag{21e}$$

$$\underset{x=b^+}{K_I}=\operatorname*{Lim}_{w\to1^+}\left\{(w-1)^{1/2}\frac{C}{\pi}\int_{-1}^{1}\frac{b_2(\phi)\,\mathrm{d}\phi}{(\phi-w)(1-\phi^2)^{1/2}}\right\}=\frac{-C}{\sqrt{2}}b_2(1) \tag{21f}$$

$$\underset{x=b^-}{K_{II}}=\operatorname*{Lim}_{w\to1^-}\left\{(1-w)^{1/2}\frac{(-C)\beta b_2(w)}{(1-w^2)^{1/2}}\right\}=\frac{-C\beta}{\sqrt{2}}b_2(1) \tag{21g}$$

where

$$b_x(x)=(1-x^2)^{1/2}B_x(x),\text{ and}$$

$b_x(\pm1)$ can be calculated from eqn (8). $b_1(\pm1)$ and $b_2(\pm1)$ can be obtained in terms of $b_1(\xi_i)$ and $b_2(\phi_i)$ respectively using Krenk's formula[14].

Three different sets of "a" and "b" were tried. Their corresponding γ's and stress intensity factors were shown in Table 1.

It is apparent from (21a,b,f,g) that

$$\underset{x=-a^-}{K_{II}}=\beta\underset{x=-a^+}{K_I} \tag{22a}$$

$$\underset{x=b^-}{K_{II}}=\beta\underset{x=b^+}{K_I}. \tag{22b}$$

Table 1. The extent of contact zone and the various K's under unit shear for three different sets of "a" and "b". ($\beta = 0.107$, $C = 0.427 \times 10^9$ Pa.)

	a = 3.0, b = 1.5	a = 1.5, b = 1.5	a = 1.5, b = 3.0
γ	0.564	0.486	0.492
K_I, $x = -a^+$	-0.578×10^{-2}	-0.203×10^{-1}	-0.203×10^{-1}
K_{II}, $x = -a^-$	-0.621×10^{-3}	-0.219×10^{-2}	-0.218×10^{-2}
K_{II}, $x = -1^+$	-0.711	-0.701	-0.702
K_I, $x = 1^-$	-0.868×10^{-1}	-0.829×10^{-1}	-0.812×10^{-1}
K_{II}, $x = 1^+$	0.706	0.706	0.707
K_I, $x = b^+$	0.790×10^{-2}	0.502×10^{-2}	0.213×10^{-2}
K_{II}, $x = b^-$	0.850×10^{-3}	0.540×10^{-3}	0.228×10^{-3}

Moreover, except for the Mode I stress intensity factor at $x = b$, the value of which is also relatively smaller than that under tensile loading, all the Mode I stress intensity factors are negative.

2. COMBINED LOADING

Due to the nature of non-linearity of this problem, the case for combined loading could not be solved by superposition of the two previous solutions for separated tensile and shear loadings. Moreover, if a compressive load is to be added to the shear field, both the right and left interlocked regions could be partially closed near the tips. To find solutions for those cases, we have to deal with the problems of additional unknown parameters that specify the sizes of the various additional contact zones. This would mean, in the context of the present numerical scheme, a considerably increased computing effort. Since we expect that the mode I stress intensity factors will be less than those for the case of pure shear loading, it is reasonable not to develop solutions for compression-shear cases, particularly if one is interested only in the worst case.

For the case of combined tensile and shear loading, we would expect, for a value of "a" and "b", γ will move to the right for increasing tension to shear ratio; finally, as γ becomes high enough, the whole crack would be opened.

In this section, we are dealing with cases for combined tensile-shear loading, where the tension/shear ratio is small enough so that the crack is still partially closed between $x = \gamma$ to 1.

The governing equations for such a case can be obtained from eqns (4) to (6) with slight modifications.

$$\tau - C\left\{\beta B_1(x)H(\gamma - x) - \frac{1}{\pi}\int_{-1}^{1}\frac{B_x(\xi)\,d\xi}{\xi - x}\right\} = 0, \quad -1 < x < 1, \tag{23a}$$

$$\sigma + C\left\{\beta B_x(x)H(x+1) + \frac{1}{\pi}\int_{-a}^{\gamma}\frac{B_1(\xi)\,d\xi}{\xi - x} + \frac{1}{\pi}\int_{1}^{b}\frac{B_2(\rho)\,d\rho}{\rho - x}\right\} = 0, \quad -a < x < \gamma, \tag{23b}$$

$$\sigma + \frac{C}{\pi}\left\{\int_{-a}^{\gamma}\frac{B_1(\xi)\,d\xi}{\xi - x} + \int_{1}^{b}\frac{B_2(\rho)\,d\rho}{\rho - x}\right\} = 0, \quad 1 < x < b, \tag{23c}$$

where σ is the applied tension at infinity.

Table 2. The size of contact zone and the stress intensity factors for combined tensile-shear loading. ($\beta = 0.107$, $C = 0.427 \times 10^9$ Pa, $\tau = 1$ Pa.)

a = 1.5, b = 1.5	σ/τ	
	0.02	0.05
γ	0.698	0.875
K_I, $x = -a^+$	-0.544×10^{-1}	-0.605×10^{-1}
K_{II}, $x = -a^-$	-0.585×10^{-2}	-0.650×10^{-2}
K_{II}, $x = -1^-$	-0.712	-0.705
K_I, $x = 1^-$	-0.104	-0.137
K_{II}, $x = 1^+$	0.704	0.701
K_I, $x = b^+$	0.235×10^{-1}	0.509×10^{-1}
K_{II}, $x = b^-$	0.252×10^{-2}	0.547×10^{-2}

Following the same procedures for solution as in the two previous sections, we found that, for $a = b = 1.5$, a σ/t ratio of as low as 0.05 can move γ from 0.486 in the pure shear case to 0.875. Thus a small σ/τ is sufficient to open up the whole crack region. Two values of σ/τ ratio were tried, namely 0.02 and 0.05. Their corresponding γ's and stress intensity factors are given in Table 2.

Discussion and summary

For an interfacial crack which is completely interlocked in the tangential direction, it can be easily shown that pure shear loading would not cause any crack opening. Crack opening can only be induced under shear if there exists within the crack zone certain smooth regions where relative slip between the dissimilar surfaces are permitted. In those situations, it was shown that the mode I and mode II stress intensity factors are related through the Dundurs' constant β. For positive β and τ, the only positive mode I intensity factor is at the right crack tip $x = b^+$. For $\beta = 1.07$, which pertains to an example of the cancellous bone/PMMA interfaces, its value is roughly 2 orders of magnitude lower than the highest mode II stress intensity factor induced. It was also noted that the highest mode II stress intensity factor occurs at the transition between the smooth and the interlocked zones and both sides of the crack. Hence, under pure shear loading, additional failure of the mode II type might occur in the regions which had already failed in the tensile direction. This fracture mode will inevitably involve cohesive fracture of either cancellous bone or the PMMA plugs along the mechanical interlocks on the interface, causing extension of the smooth zone and allowing larger relative motions between the two crack surfaces.

From the governing equations, it can be easily seen that under pure shear loading, the dimension of the contact zone γ depends only on the Dundurs' constant β and not on the applied shear τ and the other bielastic constant C. Under combined tension-shear loading, the contact region would become smaller compared with that for the pure shear case. Interestingly, the additional tensile load does not shift all mode I stress intensity factors towards the positive direction. Instead, while the K_I at $x = b^+$ becomes more positive with additional tensile load applied, K_I at $x = -a^+$ and K_I at $x = 1^-$ becomes more negative.

This paper only presents numerical analysis for one particular example of the cancellous bone/PMMA interface. Investigations with other bielastic constants will be necessary if applications are to be made for interfaces in other composite systems.

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